MAC-CPTM Situations Project

Situation 21: Exponential Rules

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Prompt

A teacher in an Algebra II college preparatory class puts the following problem on the board $x^m \bullet x^n = x^5$. She has just gotten done with her lesson which reviewed the rules for exponents. She tells her students to take out a piece of paper and write down as many answers for *m* and *n* that the students think will make the statement true. She promises the students extra credit if they write all of them down. After allowing the students a few minutes to write out answers to the problem, one student raises her hand and says aloud so the students around her can hear, "How can I ever get extra credit? I keep thinking of new one's that work." Several other students in the vicinity seem to agree.

What ideas or focal points might the teacher use to capitalize on the students comments?

Commentary:

This prompt provides a wonderful opportunity to help students deepen their understanding of exponent rules, as well as their understanding of continuous functions and the notion of infinite solutions. The following foci address each of these issues.

Mathematical Foci:

Mathematical Focus 1: Understanding why $x^m \cdot x^n = x^{m+n}$

$x^m \cdot x^n = x^{m+n}$

WHY?

x^m means x multiplied by itself m times, or $(x_1 x_2 x_3 \ldots x_m)$
x^n means x multiplied by itself n times, or $(x_1 x_2 x_3 \ldots x_n)$

Therefore $x^m \cdot x^n = (x_1 x_2 x_3 \ldots x_m)(x_1 x_2 x_3 \ldots x_n) = (x_1 x_2 x_3 \ldots x_{(m+n)})$
(that is, x multiplied by itself m times, and n more times, for a total of $(m+n)$ times, or x^{m+n}).

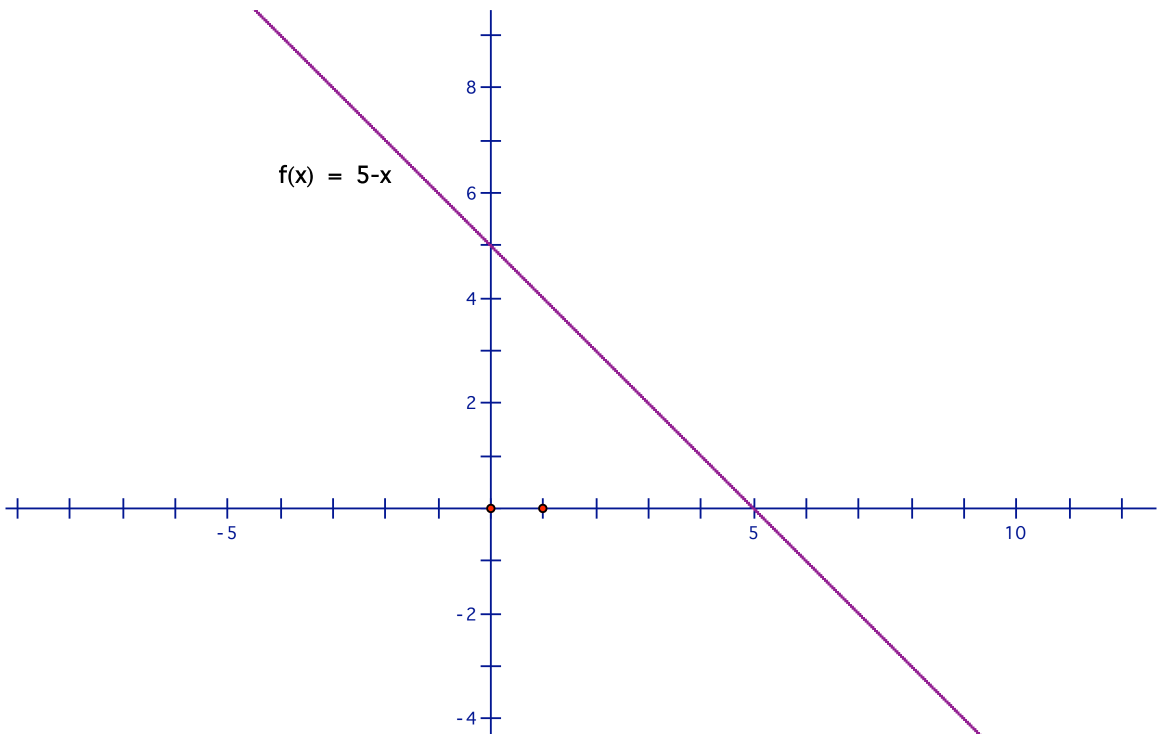
Mathematical Focus 2: $m + n = 5$ as a function

In this Situation, the specific problem is $x^m \cdot x^n = x^5$. Using the exponent rule $x^m \cdot x^n = x^{m+n}$,

$$x^m \cdot x^n = x^5$$
$$x^{m+n} = x^5$$
$$m + n = 5$$

$m + n = 5$ can be considered as a function:

$$m = f(n) = 5 - n$$



The graph shows that this is a continuous function. That is, it forms an unbroken set of points. The fact that this graph is a line (rather than a discrete number of points) indicates that there are infinite solutions to the equation $m + n = 5$.

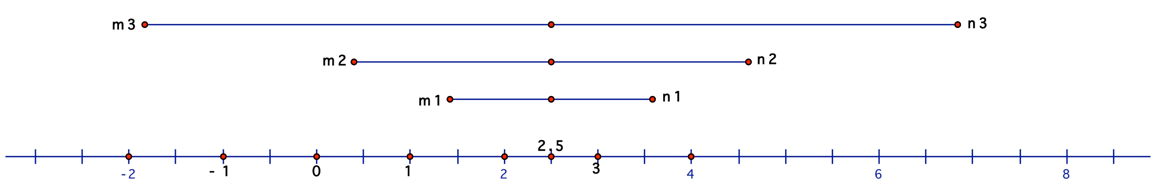
Mathematical Focus 3: Plotting *m* and *n* on a number line.

If $m + n = 5$, then the average (mean) of *m* and *n* is 2.5. That is,

$$m + n = 5$$

$$\frac{m+n}{2} = \frac{5}{2} = 2.5$$

The mean of two values is shown on a number line in this manner: A pair of values has a mean of 2.5 if each value is represented as a point that is equidistant from 2.5 on the number line. In other words, two values *m* and *n* are represented as endpoints of a segment whose midpoint is 2.5.



Each unique pair of endpoints represents a unique solution to $m + n = 5$. Since the number line extends infinitely in opposite directions, there are infinite pairs of points whose midpoint is 2.5, and therefore infinite solutions to $m + n = 5$. In the number line above (m1, n1), (m2, n2), and (m3, n3) represent a few of these infinite solutions.